Université Grenoble Alpes and ENSIMAG
Master SIAM
Stochastic modelling for neurosciences

Lab 4: Particle filter

Objectives: *Implementing a particle filter.*

Let us consider a discretized auto-regressive model for $j = 0, \dots, n$,

$$X_{j+1} = \phi X_j + U_j$$

with $U_j \sim \mathcal{N}(0, \tau^2)$ and where the observation is

$$V_j = X_j + \eta_j$$

with $\eta_j \sim \mathcal{N}(0, \sigma^2)$.

1. Simulate $n = 100$ observations with $\phi = 0.9$, $\tau^2 = 0.1$, $\sigma^2 = 1$ starting with $X_0 = -10$.
   We want to filter $X_{1:n}$ from the observations $V_{1:n}$.
2. Implement the particle filter using the transition density as proposal distribution, with $K = 100, 500, 1000, 2000$ particles. Sample one filter trajectory and compare to the true simulated X.
3. Implement the particle filter with the optimal proposal equal, at time j

$$\mathcal{N}\left(\frac{\tau^2\sigma^2}{\tau^2+\sigma^2}\left(\frac{\phi X_{j-1}}{\tau^2} + \frac{V_j}{\sigma^2}\right), \frac{\tau^2\sigma^2}{\tau^2+\sigma^2}\right)$$