Université Grenoble Alpes and ENSIMAG
Master of Science in Industrial and Applied Math (MSIAM)

Project "Stochastic modelling of neuronal data" MSIAM students

Subject 3

Elliptic Erlang memory model

Consider the stochastic system that models the memory of a neuronal network with an Erlang kernel (Ditlevsen, Locherbach, 2016)

$$\begin{aligned} dV_t &= (-\nu V_t + U_t)\, dt \\ dU_t &= (-\nu U_t + c\, f(V_t))\, dt + \sigma \sqrt{f(V_t)} dB_t \end{aligned}$$

where the variable V_t represents the membrane potential of the neuron at time t, and U_t is a recovery variable, which could represent channel kinetic. The function f is the following

$$f(x) = \begin{pmatrix} 10e^x & \text{for } x < \log(20) \\ \frac{400}{1+400e^{-2x}} & \text{for } x <\geq \log(20) \end{pmatrix} \tag{1}$$

Parameters to be estimated are $\theta = (\nu, c, \sigma)$.

Project

In your project, you will study the Erlang model by answering some of the following questions

1. Prove that the model is hypoelliptic.
2. Simulate some trajectories with an exact scheme or an approximate scheme with $\nu = 1, c = 1, \sigma = 1/4$.
3. Propose an estimation method of parameters when both coordinates V, U are observed at discrete times.
   - Bayesian approach based on Monte Carlo Markov Chain method: Pokern et al 2011
   - Minimization of the contrast defined by the Euler discretization: Samson, Thieullen 2012
   - Minimization of the contrast defined by the local linearization: Léon et al 2016.

   You will first explain the method, give the intuition, resume the principal theoretical results, and try to implement the method on simulated data.
4. Propose a method to filter the unobserved component U when only V is observed at discrete times and when parameters are known or not. Filtering can be based on

- Kalman filter: Paninski et al 2010
- Linearization and MCMC: Pokern et al 2011
- Particle filter: Paninski et al 2012, Ditlevsen and Samson 2014

You will first explain the method, give the intuition, resume the principal theoretical results if there exist, and try to implement the method on simulated data.

5. Propose an estimation method of parameters when only V is observed at discrete times. The estimation method could be

   - EM algorithm coupled with a filter: Paninski et al 2010; Paninski et al 2012; Ditlevsen and Samson 2014
   - Bayesian approach based on Monte Carlo Markov Chain method: Pokern et al 2011

   You will first explain the method, give the intuition, resume the principal theoretical results if there exist, and try to implement the method on simulated data.