Université Grenoble Alpes and ENSIMAG
Master SIAM
Stochastic modelling for neurosciences

Lab 2: estimation of one dimensional neuronal models

Objectives: *Being able to estimate parameters of some stochastic neuronal models.*

1. Wiener process with drift

$$dV(t) = Idt + \sigma dB(t), \quad V(0) = V_0 \tag{1}$$

   (a) Simulate a trajectory of $V(t)$ at discrete times $t_i = i\Delta$, $i = 1, \ldots, n$, with $\Delta = 0.0001, n = 100$ and $I = -10$, $V_0 = -65$, $\sigma = 10$.

   (b) Estimate the parameters $\theta = (I, \sigma)$ by maximum likelihood.

   (c) Repeat the simulations 100 times, and compute the means of the two estimators, and their mean squared error (MSE).

   (d) Compute the variances of the two estimators and compare with the empirical MSE. Comment.

   (e) Increase n and comment.

2. Ornstein-Uhlenbeck process

$$dV(t) = \left(-\frac{V(t) - \alpha}{\tau}\right) dt + \sigma\, dB(t), \quad V(0) = V_0, \quad \text{with } \alpha = V_0 + \tau I \tag{2}$$

   (a) Simulate a trajectory of $V(t)$ at discrete times $t_i = i\Delta$, $i = 1, \ldots, n$, with $\Delta = 0.001, n = 100$. Parameters are $\tau = 0.5$, $V_0 = -65$, $I = 50$, $\sigma = 10$.

   (b) Estimate the parameters $\theta = (\alpha, \tau, \sigma)$ by maximum likelihood.

   (c) Estimate the parameters with the Euler pseudo-likelihood.

3. Feller process

$$dV(t) = \left(-\frac{V(t) - \alpha}{\tau}\right) dt + \sigma\sqrt{V(t) - V_I}\, dB(t), \quad V(0) = V_0 \tag{3}$$

   (a) Simulate a trajectory of $V(t)$ at discrete times $t_i = i\Delta$, $i = 1, \ldots, n$, with $\Delta = 0.001, n = 5000$. Parameters are $\tau = 0.5$, $V_0 = -65$, $I = 50$, $V_I = -70$, $\sigma = 10$.

   (b) Estimate the parameter μ with the least squares and the conditional least squares methods.

   (c) Repeat the simulations and compare the accuracy of both estimators.