Université Joseph Fourier and ENSIMAG
Master SIAM
Stochastic modelling for neurosciences

Lab 1: simulation of neuronal models

Objectives: *Being able to simulate any neuronal model, with exact or approximate numerical schemes.*

1 Exact simulation of one-dimensional neuronal model

1. Wiener process with drift

$$dV(t) = Idt + \sigma dB(t), \quad V(0) = V_0 \tag{1}$$

   (a) Compute the exact distribution of $(V(t))$.

   (b) Simulate a trajectory of $V(t)$ at discrete times $t_i = i\Delta$, $i = 1, \dots, n$, with $\Delta = 0.0001, n = 1000$ and $I = -10$, $V_0 = -65$, $\sigma = 1$.

   (c) Change the value of the diffusion coefficient: $\sigma = 10$ or $\sigma = 20$. Comment.

2. Ornstein-Uhlenbeck process

$$dV(t) = \left(-\frac{V(t) - \alpha}{\tau}\right) dt + \sigma\, dB(t), \quad V(0) = V_0, \quad \text{with } \alpha = V_0 + \tau I \tag{2}$$

   (a) Compute the exact distribution of $(V(t))$.

   (b) Simulate a trajectory of $V(t)$ at discrete times $t_i = i\Delta$, $i = 1, \dots, n$, with $\Delta = 0.001, n = 5000$. Parameters are $\tau = 0.5$, $V_0 = -65$, $I = 50$, $\sigma = 10$.

   (c) Compute the spiking times with the threshold $S = -45$.

   (d) Plot the distribution of the spiking times. Plot the membrane potential evolution.

   (e) Change the value of the input $I = 20$. Comment.

2 Approximate simulation of one-dimensional neuronal model

1. Ornstein-Uhlenbeck process

   (a) Write the Euler scheme for the OU process (2) and implement the scheme.

   (b) Compare the trajectories obtained with the exact scheme, depending on the value of Δ.

2. Feller process

$$dV(t) = \left(-\frac{V(t) - \alpha}{\tau}\right) dt + \sigma\sqrt{V(t) - V_I}\, dB(t), \quad V(0) = V_0 \tag{3}$$

   (a) Write the the Euler scheme for the Feller process (3) and implement the scheme.

   (b) Simulate a trajectory of $V(t)$ at discrete times $t_i = i\Delta$, $i = 1, \dots, n$, with $\Delta = 0.001, n = 5000$. Parameters are $\tau = 0.5$, $V_0 = -65$, $I = 50$, $V_I = -70$, $\sigma = 10$.

(c) Compute the spiking times with the threshold $S = -45$.

(d) Plot the distribution of the spiking times.

(e) Change the value of V_I. Comment.

3 Approximate simulation of multi-dimensional neuronal model

1. Elliptic FitzHugh Nagumo

$$\begin{cases} dV_t &= \frac{1}{\varepsilon}(V_t - V_t^3 - U_t - s)dt + \sigma_1 dB_t^1, \\ dU_t &= (\gamma V_t - U_t + \beta)\,dt + \sigma_2 dB_t^2, \end{cases} \tag{4}$$

where the variable V_t represents the membrane potential of the neuron at time t, and U_t represents the channel kinetic. Parameter s is the magnitude of the stimulus current.

(a) Write the Euler scheme for the FitzHugh Nagumo process and implement the scheme.

(b) Simlate trajectories with the following parameters : $\varepsilon = 0.1$, $s = 0$, $\gamma = 1.5$, $\beta = 0.8$, $\sigma_1 = 0.3$, $\sigma_2 = 0.3$. The time step is $\delta = 0.02$. Simulate a trajectory of length $n = 1000$.

(c) Simlate trajectories for the hypoelliptic system with the following parameters : $\varepsilon = 0.1$, $s = 0$, $\gamma = 1.5$, $\beta = 0.8$, $\sigma_1 = 0$, $\sigma_2 = 0.3$. The time step is $\delta = 0.02$. Simulate a trajectory of length $n = 1000$.

2. Morris-Lecar process

$$\begin{aligned} dV(t) &= -\left(g_{fast}\, m_\infty(t)\,(V(t) - V_{fast}) + g_{slow}U(t)(V(t) - V_{slow}) + g_L(V(t) - V_L) + I(t)\right) dt + +\gamma d\tilde{B}(t) \\ dU(t) &= -\frac{1}{\tau_u(V(t))}(U(t) - u_0(V(t))) = (\alpha(V(t))(1 - U(t)) - \beta(V(t))U(t))\,dt + \sigma(V(t), U(t))dB(t) \end{aligned}$$

with

$$\begin{aligned} m_\infty(v) &= \frac{1}{2}\left(1 + \tanh(\frac{v - V_1}{V_2})\right) \\ u_0(v) &= \frac{1}{2}\left(1 + \tanh(\frac{v - V_3}{V_4})\right) \\ \tau_u(v) &= \frac{\tau_u}{\cosh\left(\frac{v-V_3}{V_4}\right)} \\ \alpha(v) &= \frac{1}{2}\phi\cosh\left(\frac{v - V_3}{2V_4}\right)\left(1 + \tanh\left(\frac{v - V_3}{V_4}\right)\right), \\ \beta(v) &= \frac{1}{2}\phi\cosh\left(\frac{v - V_3}{2V_4}\right)\left(1 - \tanh\left(\frac{v - V_3}{V_4}\right)\right) \\ \sigma(v, u) &= \sigma\sqrt{2\frac{\alpha(v)\beta(v)}{\alpha(v) + \beta(v)}u(1 - u)}. \end{aligned}$$

(a) Write the Euler scheme for the Morris-Lecar process and implement the scheme.

(b) Simlate trajectories with the following parameters $V_{fast} = -84; V_L = -60; V_{slow} = 120; I = 88/20; g_L = 2/20; g_{slow} = 4.4/20; g_{fast} = 8/20; V_1 = -1.2; V_2 = 18; V_3 = 2; V_4 = 30; \phi = 0.04; \sigma = 0.03; \gamma = 1$. The initial conditions are $V_0 = -26; U_0 = 0.2$. The time step is $\delta = 0.1$. Simulate a trajectory of length $n = 5000$.